2] Write down the conditions (Group postulates) for the symmetry elements to form a mathematical group.

A set of elements with a binary operation(*) form a group if the elements in the set obey the following rules.

Rule : 1

The product of any two elements A and B in the group combine to give the third element C, which is also an element of the group.and square of each element must be an element in the group.

*A*xB = C

Rule :2

An element combines with itself to form another element of the group.

AxA = E

Here E is the member of the same group.

Rule: 3

One element in the group must commute with all others and leave them unchanged. It is designed by E and it is usually represented as identity element.

For example,

AE = EA = A

BE = EB=B (here E is the identity element)

Rule : 4

Every element A of a group has an inverse A^{-1} which is also an element of the group.

$A \times A^{-1} = A^{-1} \times A = E$

Therefore, A^{-1} is the inverse of element A.

Similiarly, A^{-1} , A and E should belong to the group G.

Rule : 5

Every element of the group obeys the associative law of combination.

A (BC) = (AB)C

....

3) Define the following terms:

a) Abelian groups:

A Group is said to be abelian, if all the elements commute with each other.

Example : H_2O molecule belongs to Abelian Group.

b) Non-abelian groups:

A Group is said to be Non-abelian, if all the elements do not commute with each other.

Example : NH_3 molecule belongs to Non-abelian Group.

c) cyclic groups:

A group is said to be cyclic, if all its elements can be generated from the symmetry element. Thus A, A^2, A^3, A^n form the elements of the cyclic group.

Here $A^n = E$, the identity element.

In general, the roots of the equation $x^n - 1 = 0$ form a cyclic group.

d) Order of the group:

The total number of elements of a group is called as order of the group.

For example,

1.Water molecule C_{2V} point group.

The elements are $(E, C_2, \sigma_v', \sigma_v'')$

The order of the group is 4.

2. Ammonia molecule C_{3V} point group.

The elements are $(E, C_3, C_3^2, \sigma_v', \sigma_v'', \sigma_v''')$

The order of this group is 6.

e) sub-group:

This is a smaller group within a group. If any selection or subset of the element of a group satisfies the definition of a group, then this subset of the element is called a sub-group.

*****Identity element is essentially a part of sub-group.

Example ; $\{E, C_2\}$ is a sub-group of C_{2V} point group.

4} Construct the Group Multiplication table for C_{2V} point group

A water molecule has four elements, $(E, C_2, \sigma_v', \sigma_v'')$

We can easily show that the product of any two symmetry elements is one of the four elements of the group.

Thus for instance, $C_2 \times \sigma_v' = \sigma_v''$

Proceeding this way the symmetry operations of H_2O molecule can be listed in a Group Multiplication table.

Step : 1

Water molecule has 4 symmetry elements . Hence, they are arranged in 4x4 table as follows.

	E	C_2	$\sigma_v^{'}$	$\sigma_v^{''}$
E				
C_2				
$\sigma_v^{'}$				
$\sigma_v^{''}$				

Step :2

Multiply all the symmetry elements of water molecule by E

$E x E = E$

$E x C_2 = C_2$

$E x\ \sigma_v^{'} = \sigma_v^{'}$

$E x\ \sigma_v^{'} = \sigma_v^{''}$

Now the Group multiplication table is filled as follows

	E	C_2	$\sigma_v^{'}$	$\sigma_v^{''}$
E	E	C_2	$\sigma_v^{'}$	$\sigma_v^{''}$
C_2				
$\sigma_v^{'}$				
$\sigma_v^{''}$				

Step :3

Multiply all the symmetry elements of water molecule by C_2

$C_2 x E = C_2$

$C_2 x C_2 = E$

$C_2 x\ \sigma_v^{'} = \sigma_v^{''}$

$C_2 x\ \sigma_v^{''} = \sigma_v^{'}$

Now the Group multiplication table at the end of step:3 is filled as follows

	E	C_2	σ_v'	σ_v''
E	E	C_2	σ_v'	σ_v''
C_2	C_2	E	σ_v''	σ_v'
σ_v'				
σ_v''				

Step :4

Multiply all the symmetry elements of water molecule by σ_v'

$\sigma_v' x E = \sigma_v'$

$\sigma_v' x C_2 = \sigma_v''$

$\sigma_v' x\ \sigma_v' = E$

$\sigma_v' x\ \sigma_v'' = C_2$

Now the group multiplication table at the end of step 4 is

	E	C_2	σ_v'	σ_v''
E	E	C_2	σ_v'	σ_v''
C_2	C_2	E	σ_v''	σ_v'
σ_v'	σ_v'	σ_v''	E	C_2
σ_v''				

Step :5

Multiply all the symmetry elements of water molecule by σ_v''

$\sigma_v'' x E = \sigma_v''$

$\sigma_v'' x C_2 = \sigma_v'$

$\sigma_v'' x\ \sigma_v' = C_2$

$\sigma_v'' x\ \sigma_v'' = E$

Now the group multiplication table at the end of step 5 is

	E	C_2	σ_v'	σ_v''
E	E	C_2	σ_v'	σ_v''
C_2	C_2	E	σ_v''	σ_v'
σ_v'	σ_v'	σ_v''	E	C_2
σ_v''	σ_v''	σ_v'	C_2	E